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DEFORMABLE WING OF HIGH ASPECT RATIO IN A BOUNDED FLUID (DEFORM--ETC(U)
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DEFORMABLE WING OF HIGH ASPECT RATIO IN A BOUNDED FLUID

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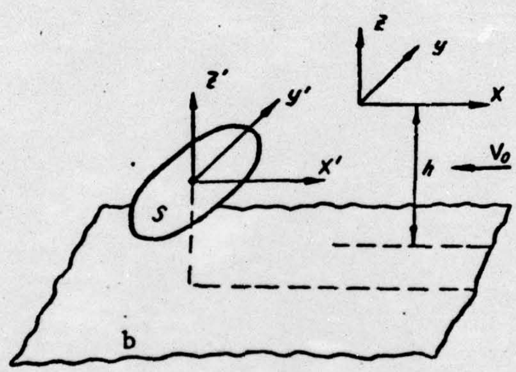
The aerodynamic characteristics of a lifting element are generally determined by a set of parameters /12*

$$X = X(\bar{h}, \lambda, Fr, M, P, H, Re, \ldots).$$

The group of hydroaerodynamic problems involving studies of characteristics with a small number of parameters (one or two) has now been adequately investigated. Problems involving many parameters are represented by a narrower range of studies.

The present paper, which deals with a three-parameter case, investigates the lift effectiveness of a thin deformable (H) surface (λ) in a bounded fluid ($\bar{h}$).



§1. Let us consider a deformable lifting surface S, moving steadily at velocity V_0 in an ideal incompressible fluid near some boundary "b" at small local angles of attack. We introduce the right-handed coordinate system XYZ, fixed in the surface S, with the X axis parallel to the undisturbed boundary at a distance h from the latter and the Z axis pointing vertically upward (see figure).

In the linear formulation, the problem amounts to solving the equation of a deformable lifting surface,[5,4] having the following form for limiting Froude numbers:

$$\iint_S \gamma(\zeta, \eta)\left\{\frac{d}{dy}\frac{1}{y-\eta}\left[1-\frac{\sqrt{(x-\xi)^2+(y-\eta)^2}}{x-\xi}\right]+\frac{F}{[(y-\eta)^2+4h^2]^2}\times\right.$$
$$\times\left\{\frac{(x-\xi)[(y-\eta)^2<(x-\xi)^2+(y-\eta)^2>-4h^2<(x-\xi)^2+(y-h)^2+8h^2>]}{\sqrt{(x-\xi)^2+(y-\eta)^2+4h^2}^3}-\right.$$
$$\left.\left.-[(y-\eta)^2-4h^2]\right\}\right\}d\zeta d\eta = V_0[f_{ox}(x, y)+\iint_S C_x(x, y, \xi, \eta)\times$$
$$\times p(\xi, \eta)\,d\xi d\eta]. \qquad (1)$$

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*Numbers in the right margin indicate pagination in the original text.

where $\gamma(\xi, \eta)$ is the density of doublets; /13

$p(\xi, \eta) = \rho V_0 \gamma(\xi, \eta)$ is the distributed load;

$C(x, y, \xi, \eta)$ is a two-dimensional function of the deformability effect.

In the monoaxisymmetric case, we introduce the following transformation of coordinates:

$$y = y' + \frac{y_1 + y_2}{2} = y' + \frac{y_{01} + y_{02}}{2}; \quad y' \in \left(-\frac{y_2 - y_1}{2}, \frac{y_2 - y_1}{2}\right); \; \bar{y}' = \frac{y'}{b}$$

and similarly for η; here $y_{01} \leqslant y, \eta \leqslant y_{02}$; y_{01}, y_{02} are the coordinates of the wing tips.

$$x(y) = x'(y) + \frac{x_1(y) + x_2(y)}{2}; \; x' \in [-a(y), a(y)]; \; \bar{x}' = \frac{x'}{b};$$

and the same for $\xi(\eta)$. Here $x_1(y) \leqslant x \leqslant x_2(y)$; $\xi_1(\eta) \leqslant \xi \leqslant \xi_2(\eta)$; $x_1(y)$ is the equation of the trailing edge; $x_2(y)$ is the equation of the leading edge.

We thus obtain a dimensionless form of the equation of a monoaxisymmetric lifting surface in a bounded fluid in an interpretation suitable for an examination of high aspect ratios:

$$\begin{gathered}
\frac{1}{2\pi}\int_{-1}^{+1}\int_{-1}^{+1}\bar{\gamma}(\bar{\xi}', \bar{\eta}')\Bigl\{\frac{d}{d\bar{y}'}\frac{1}{\bar{y}'-\bar{\eta}'}\Bigl[1-\frac{\sqrt{\Bigl(\bar{x}'+\frac{x_2+x_1}{x_2-x_1}-\bar{\xi}'-\frac{\xi_2+\xi_1}{x_2-x_1}\Bigr)^2+}}{\bar{x}+\frac{x_2+x_1}{x_2-x_1}-\bar{\xi}'-} \\
\frac{\overline{+\lambda^2(y)(\bar{y}'-\bar{\eta}')^2}}{-\frac{\xi_2+\xi_1}{x_2-x_1}}\Bigr]+\frac{F}{[(\bar{y}'-\bar{\eta}')^2+16\tilde{h}^2]^2}\Bigl\{\frac{\Bigl(\bar{x}'+\frac{x_2+x_1}{x_2-x_1}-\bar{\xi}'-\frac{\xi_2+\xi_1}{x_2-x_1}\Bigr)\times}{\lambda(y)\sqrt{\frac{1}{\lambda^2(y)}\Bigl(\bar{x}'+\frac{x_2+x_1}{x_2-x_1}-}} \\
\frac{\times\Bigl[(\bar{y}'-\eta')^2<\frac{1}{\lambda^2(\bar{y})}\Bigl(\bar{x}'+\frac{x_2+x_1}{x_2-x_1}-\bar{\xi}'-\frac{\xi_2+\xi_1}{x_2-x_1}\Bigr)^2+(\bar{y}'-\bar{\eta}')^2>-16\tilde{h}^2<}{\overline{-\bar{\xi}'-\frac{\xi_2+\xi_1}{x_2-x_1}\Bigr)^2+(\bar{y}'-\bar{\eta}')^2+16\tilde{h}^2}^{\,3}} \\
\frac{<\frac{1}{\lambda^2(y)}\Bigl(\bar{x}'+\frac{x_2+x_1}{x_2-x_1}-\bar{\xi}'-\frac{\xi_2+\xi_1}{x_2-x_1}\Bigr)^2+(\bar{y}'-\bar{\eta}')^2+32\tilde{h}^2>\Bigr]}{}\times \\
\times\Bigl[(\bar{y}')-\bar{\eta}')^2-16\tilde{h}^2\Bigr]\Bigr\}\Bigr\}\, d\bar{x}\, d\bar{y}=\frac{1}{a(y)}\Bigl|f_{0\bar{x}'}(\bar{x}', \bar{y}')+2Ab^2\int_{-1}^{+1}\int_{-1}^{+1}C_{\bar{x}'}\times \\
\times(\bar{x}', \bar{y}, \bar{\xi}', \bar{\eta}')\,\bar{\gamma}(\bar{\xi}', \bar{\eta}')\,d\bar{\xi}'d\bar{\eta}'], \qquad (2)
\end{gathered}$$

where

$\bar{\gamma}(\bar{\xi}', \bar{\eta}') = \frac{\gamma(\bar{\xi}', \bar{\eta}')}{2\lambda(y)V_0}$, $\lambda = \frac{b}{a(y)}$ is the aspect ratio;

b is the semispan;

$a(y)$ is the half-chord;

$\tilde{h} = \frac{h}{2b} = \frac{h}{\ell} = \frac{\bar{h}}{\lambda(y)}$ is the relative distance along the span; /14

ℓ is the span;

$\bar{h}$ is the relative distance along the chord;

$f_0(\bar{x}', \bar{y}')$ is the form equation of the initial undeformed surface in dimensionless coordinates;

$f_{0\bar{x}'}$ is the derivative of the form equation;

$A = \rho V_0^2$;

$$F = \begin{cases} 0 \text{ in an unbounded fluid} & \\ +1 \text{ under a free surface} & \sim F_r = \infty \\ -1 \text{ above a screen.} & \sim F_r = 0 \end{cases}$$

§2. Because of the impossibility of an analytical solution of the two-dimensional Equation (2), limitations will be introduced for arbitrary planforms: a wing of high aspect ratio will be considered. Then, assuming $\frac{x_2 + x_1}{2} \approx \frac{\xi_1 + \xi_1}{2}$ and using the approximation

$$\sqrt{\left(\bar{x}' + \frac{x_2+x_1}{x_2-x_1} - \bar{\xi}' - \frac{\xi_2+\xi_1}{x_2-x_1}\right)^2 + \lambda^2(y)(\bar{y}' - \bar{\eta}')^2} \approx \lambda(y)\,|\bar{y}' - \bar{\eta}'|,$$

we obtain from Eq. (2)

$$\frac{1}{2\pi}\int_{-1}^{+1}\int_{-1}^{+1}\bar{\gamma}(\bar{\xi}', \bar{\eta}')\left\{\frac{d}{d\bar{y}'}\frac{1}{\bar{y}'-\bar{\eta}'}\left[1 - \frac{\lambda(y)|\bar{y}'-\bar{\eta}'|}{\bar{x}-\bar{\xi}'}\right] - F\frac{(\bar{y}'-\bar{\eta}')^2 - 16\tilde{h}^2}{[(\bar{y}'-\bar{\eta}')^2 + 16\tilde{h}^2]^2}\right\}\times$$
$$\times\, d\bar{\xi}'d\bar{\eta}' = \frac{1}{a(y)}\left[f_{0\bar{x}'}(\bar{x}', \bar{y}') + 2Ab^2\int_{-1}^{+1}\int_{-1}^{+1} C_{\bar{x}'}(\bar{x}', \bar{y}', \bar{\xi}', \bar{\eta}')\times\right.$$
$$\left.\times\,\bar{\gamma}(\bar{\xi}', \bar{\eta}')\,d\bar{\xi}'d\bar{\eta}'\right]. \qquad (3)$$

Assuming $\bar{\gamma}(\bar{\xi}, \bar{\eta}') = \bar{\gamma}(\bar{\xi}')\bar{\gamma}(\bar{\eta}')$, we transform Eq. (3) to

$$-\frac{\lambda(y)}{2\pi}\int_{-1}^{+1}\frac{\bar{\gamma}(\bar{\xi}')}{\bar{x}'-\bar{\xi}'}\frac{d}{d\bar{y}'}\int_{-1}^{+1}\bar{\gamma}(\bar{\eta}')\frac{|\bar{y}'-\bar{\eta}'|}{\bar{y}'-\bar{\eta}'}d\bar{\eta}'d\bar{\xi}' = \frac{1}{a(y)}\left[f_{0\bar{x}}(\bar{x}', \bar{y}') + \right.$$
$$\left. + 2Ab^2\int_{-1}^{+1}\gamma(\bar{\eta}')\int_{-1}^{+1}\bar{\gamma}(\bar{\xi}')C_{\bar{x}'}(\bar{x}', \bar{y}', \bar{\xi}', \bar{\eta}')\,d\bar{\xi}'d\bar{\eta}'\right] -$$
$$-\frac{1}{2\pi}\int_{-1}^{+1}\gamma(\bar{\xi}')\int_{-1}^{+1}\bar{\gamma}(\bar{\eta}')\frac{d}{d\bar{y}'}\frac{1}{\bar{y}'-\bar{\eta}'}d\bar{\eta}'d\bar{\xi}' + F\frac{1}{2\pi}\times$$
$$\times\int_{-1}^{+1}\bar{\gamma}(\bar{\xi}')\int_{-1}^{+1}\bar{\gamma}(\bar{\eta}')\frac{[(\bar{y}'-\bar{\eta}')^2 - 16\tilde{h}^2]}{[(\bar{y}'-\bar{\eta}')^2 + 16\tilde{h}^2]^2}d\bar{\eta}'d\bar{\xi}'.$$

Considering

$$\frac{d}{d\bar{y}'}\int_{-1}^{+1}\bar{\gamma}(\bar{\eta}')\frac{|\bar{y}'-\bar{\eta}'|}{\bar{y}'-\bar{\eta}'}d\bar{\eta}' = 2\bar{\gamma}(\bar{y}')$$

and integrating by parts, providing that $\bar{\gamma}(\bar{\eta}')|_{\bar{\eta}'=\pm 1}=0$, when /15

$$\int_{-1}^{+1}\bar{\gamma}(\bar{\eta}')\frac{d}{d\bar{y}'}\frac{1}{\bar{y}'-\bar{\eta}'}d\bar{\eta}'=\int_{-1}^{+1}\bar{\gamma}_{\bar{\eta}'}(\bar{\eta}')\frac{d\bar{\eta}'}{\bar{y}'-\bar{\eta}'};\ \int_{-1}^{+1}\bar{\gamma}(\bar{\eta}')\frac{[(\bar{y}'-\bar{\eta}')^2-16\bar{h}^2]}{[(\bar{y}'-\bar{\eta}')^2+16\bar{h}^2]^2}\eta'=$$

$$=-\int_{-1}^{+1}\bar{\gamma}_{\bar{\eta}'}(\bar{\eta}')\frac{\bar{y}'-\bar{\eta}'}{(\bar{y}'-\bar{\eta}')^2+16\bar{h}^2}d\bar{\eta}',$$

we have

$$-\frac{2\lambda(y)\bar{\gamma}(\bar{y}')}{2\pi}\int_{-1}^{+1}\frac{\bar{\gamma}(\bar{\xi}')d\bar{\xi}'}{\bar{x}'-\bar{\xi}'}=\frac{1}{a(y)}\Big[f_{0\bar{x}'}(\bar{x}',\bar{y}')+$$

$$+2Ab^2\int_{-1}^{+1}\bar{\gamma}(\bar{\eta}')\int_{-1}^{+1}\bar{\gamma}(\bar{\xi}')C_{x'}(\bar{x}',\bar{y}',\xi',\bar{\eta}')d\bar{\xi}'d\bar{\eta}'\Big]-$$

$$-\frac{1}{2\pi}\int_{-1}^{+1}\bar{\gamma}(\bar{\xi}')\int_{-1}^{+1}\bar{\gamma}_{\bar{\eta}'}(\bar{\eta}')\frac{d\bar{\eta}'}{\bar{y}'-\bar{\eta}'}d\bar{\xi}'-F\frac{1}{2\pi}\int_{-1}^{+1}\bar{\gamma}(\bar{\xi}')\int_{-1}^{+1}\bar{\gamma}_{\bar{\eta}'}(\bar{\eta}')\frac{\bar{y}'-\bar{\eta}'}{(\bar{y}'-\bar{\eta}')^2+16\bar{h}^2}\times$$

$$\times d\bar{\eta}'d\bar{\xi}'. \qquad (4)$$

After integrating (4) over the chord with a weight factor $\sqrt{\frac{1-\bar{x}'}{1+\bar{x}'}}$ and introducing dimensionless circulation

$$\bar{\Gamma}(\bar{\eta}')=\bar{\gamma}(\bar{\eta}')\int_{-1}^{+1}\bar{\gamma}(\bar{\xi}')d\bar{\xi}'=\frac{\Gamma(\bar{\eta}')}{2bV_0} \qquad (5)$$

we obtain an equation for a high-aspect-ratio deformable wing in a bounded fluid, of the same form as the Prandtl equation:

$$\bar{\Gamma}(\bar{y}')=\frac{a_\infty\psi}{2\lambda(y)}\left\{\alpha(\bar{x}',\bar{y}')-\frac{1}{2\pi}\int_{-1}^{+1}\bar{\Gamma}_{\bar{\eta}'}(\bar{\eta}')\left[\frac{1}{\bar{y}'-\bar{\eta}'}+FG(\bar{y}'-\bar{\eta}')\right]d\bar{\eta}'\right\}, \qquad (6)$$

where

$$\alpha(\bar{x}',\bar{y}')=\frac{\bar{\alpha}(\bar{x}',\bar{y}')}{a(y)}=\frac{\bar{\alpha}(\bar{x}',\bar{y}')}{\pi a(y)},\quad G(\bar{y}'-\eta')=\frac{\bar{y}'-\bar{\eta}'}{(\bar{y}'-\bar{\eta}')^2+16\bar{h}^2};$$

$$\bar{\alpha}(\bar{x}',\bar{y}')=\int_{-1}^{+1}\sqrt{\frac{1-\bar{x}'}{1+\bar{x}'}}f_{0\bar{x}'}(\bar{x}',\bar{y}')d\bar{x}'+2Ab^2\int_{-1}^{+1}\bar{\Gamma}(\bar{\eta}')\times$$

$$\times\frac{\int_{-1}^{+1}\sqrt{\frac{1-\bar{x}'}{1+\bar{x}'}}\int_{-1}^{+1}\bar{\gamma}(\bar{\xi}')C_{\bar{x}'}(\bar{x}',\bar{y}',\bar{\xi}',\bar{\eta}')d\bar{\xi}'d\bar{x}'}{\int_{-1}^{+1}\bar{\gamma}(\bar{\xi}')d\bar{\xi}'}d\bar{\eta}'. \qquad (7)$$

Assuming the independence of the deformations of the cross sections, we obtain /16

$$C_{\bar{x}'}(\bar{x}',\bar{y}',\bar{\xi}',\bar{\eta}')\rightarrow C_{\bar{x}}(\bar{x}',\bar{\xi}').$$

In the case of an elastically deformable axisymmetric wing, for an arbitrary position of the projection S_p of surface S on the XOY plane, with EJ_y = const, we have[3]

$$x > \xi;\ C'_{\bar{x}'}(\bar{x}', \bar{\xi}') = \operatorname{sign} R \frac{a^3(y)}{EJ(y)} \left\{\left[\bar{x}' + \frac{x_1 + x_2}{a(y)}\right]\left[\bar{\xi}' + \frac{\xi_1 + \xi_2}{a(y)}\right] - \right.$$

$$\left. -\frac{1}{2}\left[\bar{x}' + \frac{x_1 + x_2}{a(y)}\right]^2 - \left[\bar{\xi}' + \frac{\xi_1 + \xi_2}{a(y)} + \frac{1}{2}\right]\right\}; \tag{8}$$

$$x < \xi;\ C''_{\bar{x}'}(\bar{x}', \bar{\xi}') = \operatorname{sign} R \frac{a^3(y)}{EJ_y} \left\{\frac{1}{2}\left[\bar{\xi}' + \frac{\xi_1 + \xi_2}{a(y)}\right]^2 - \left[\bar{\xi}' + \frac{\xi_1 + \xi_2}{a(y)}\right] + \frac{1}{2}\right\},$$

where sign R = -1 corresponds to the built-in end along the leading edge, and sign R = 1 to the built-in end along the trailing edge.

For a diaxisymmetric wing with a coordinate system located at the center of projection S_p, formulas (8) become simplified:

$$C'_{\bar{x}'}(\bar{x}', \bar{\xi}') = \operatorname{sign} R \frac{a^3(y)}{EJ_y}\left[\left(\bar{x}'\bar{\xi}' - \frac{1}{2}\bar{x}'^2\right) - \left(\bar{\xi}' - \frac{1}{2}\right)\right];$$

$$C''_{\bar{x}'}(\bar{x}', \bar{\xi}') = \operatorname{sign} R \frac{a^3(y)}{EJ_y}\left(\frac{1}{2}\bar{\xi}'^2 - \bar{\xi}' + \frac{1}{2}\right). \tag{9}$$

§3. Let us consider a diaxisymmetric elastic wing, planar in the initial undeformed state: $f_{0x} = \alpha_0$. Then $f_{0\bar{x}'} = \alpha_0 a(y)$, which gives

$$\tilde{a} = \alpha_0 a(y)\pi + \operatorname{sign} R \frac{2Ab^2a^2(y)}{EJ_y}\bar{A}_1 \int_{-1}^{+1} \bar{\Gamma}(\bar{\eta})\, d\bar{\eta}, \tag{10}$$

where

$$\bar{A}_1 = \frac{\int_{-1}^{+1} \sqrt{\frac{1-\bar{x}}{1+\bar{x}}}\left\{\int_{-1}^{\bar{x}} \bar{\gamma}(\bar{\xi})\, C'_{\bar{x}}(\bar{x}, \bar{\xi})\, d\bar{\xi} + \int_{\bar{x}}^{1} \bar{\gamma}(\bar{\xi})\, C''_{\bar{x}}(\bar{x}, \bar{\xi})\, d\bar{\xi}\right\} d\bar{x}}{\int_{-1}^{+1} \bar{\gamma}(\bar{\xi})\, d\bar{\xi}}.$$

If $\bar{\gamma}(\bar{\xi}) = B\sqrt{\frac{1+\bar{\xi}}{1-\bar{\xi}}}$, then $\bar{A}_1 \approx 0.75$.

Thus,

$$\tilde{a} = a(y)\pi\left[\alpha_0 + \operatorname{sign} R \frac{\bar{A}_1 HC_y(y)}{\pi}\right] = a(y)\pi(\alpha_0 + \alpha_p), \tag{11}$$

where

$$\alpha_p = \operatorname{sign} R \frac{\bar{A}_1 HC_y(y)}{\pi}; \tag{12}$$

$$C_y(y) = \lambda(y)\int_{-1}^{+1} \bar{\Gamma}(\bar{\eta})\, d\bar{\eta}. \tag{13}$$

$H = \frac{\rho V_0^2 k^3 b}{8EJ_y}$ is the similarity parameter of a statically aerohydroelastic lifting surface.

In this case, Eq. (6) becomes /17

$$\bar{\Gamma}(\bar{y}) = \frac{a_\infty b}{2\lambda(y)}\left\{\alpha_0 + \operatorname{sign} R \frac{\bar{A}_1 HC_y(y)}{\pi} - \frac{1}{2\pi}\int_{-1}^{+1} \bar{\Gamma}_{\bar{\eta}}(\bar{\eta})\left[\frac{1}{\bar{y}-\bar{\eta}} + FG(\bar{y}-\bar{\eta})\right] d\bar{\eta}\right\}. \tag{14}$$

We seek the solution of Eq. (14) in the form

$$\bar{\Gamma}(\bar{y}) = \alpha A_2 \Gamma_0(\bar{y}). \qquad (15)$$

where $\Gamma_0(\bar{y}) = \sqrt{1-\bar{y}^2}$, $A_2 = \text{const}$.

Introducing (15) into Eq. (14) and integrating the right- and left-hand parts of the latter equation over the span, we obtain

$$A_2 = \frac{\dfrac{a_\infty \psi}{\lambda}}{\displaystyle\int_{-1}^{+1} \Gamma_0(\bar{y})\, d\bar{y} + \frac{a_\infty \psi}{4\pi\lambda} \int_{-1}^{+1}\int_{-1}^{+1} \Gamma_0'(\bar{\eta}) \left[\frac{1}{\bar{y}-\bar{\eta}} + FG(\bar{y}-\bar{\eta})\right] d\bar{\eta}\, d\bar{y}}. \qquad (16)$$

To calculate A_2, we expand the regular part of the kernel[5] of the inner integral in the double integral of the denominator of (16) as a series in even powers of the small parameter $\tau_2 = \sqrt{4\bar{h}^2 + 1} - 2\bar{h}$:

$$G(\bar{y}-\bar{\eta}) = \sum_{m=2,4\ldots}^{\infty} \sum_{n=0}^{\frac{m}{2}-1} \tau_2^m \frac{(m-1-n)!\,(-1)^{\frac{m}{2}-n+1}}{n!\,(m-1-2n)!} (\bar{y}-\bar{\eta})^{m-1-2n}.$$

Keeping terms up to τ_2^6 inclusive in the expansion, we have

$$A_2 = \frac{\dfrac{a_\infty \psi(\bar{h})}{\lambda}}{\dfrac{\pi}{2} + \dfrac{a_\infty \psi(\bar{h})}{2\lambda} \zeta(\bar{h})}, \qquad (17)$$

where

$$a_\infty = \frac{dC_{y\infty}}{d\alpha};$$

$C_{y\bar{h}}$ is the lift coefficient of the thin airfoil near the boundary;

$C_{y\infty}$ is the lift coefficient of the thin airfoil in an unbounded fluid;

$\psi(\bar{h}) = \dfrac{a_{\bar{h}}}{a_\infty}$ is the function of the boundary effect in a two-dimensional problem;

$$\zeta(\bar{h}) = 1 + F\frac{1}{2}\left(\tau_2^2 + \frac{1}{4}\tau_2^4 + \frac{1}{8}\tau_2^6\right) \qquad (18)$$

is a function allowing for the effect of finiteness of the span near the boundary.

Thus, using Eq. (13), we arrive at an equation for the lift coefficient of the wing

$$C_y = \frac{\pi\lambda A_2}{2}\left(\alpha_0 + \operatorname{sign} RH + \frac{\bar{\lambda}_1 C_y}{\pi}\right). \qquad (19)$$

which results in /18

$$C_y = C_{y_{nd}} \psi(\bar{h}, \lambda, H), \qquad (20)$$

where $C_{y_{nd}}$ is the lift coefficient of a planar nondeformable wing of high aspect ratio in a bounded fluid;[5]

$$C_{y_{nd}} = \frac{a_\infty \psi(\bar{h})}{1 + \frac{a_\infty \psi(\bar{h})}{\pi\lambda}\zeta}\alpha_0; \tag{21}$$

$\psi(\bar{h})$ is the boundary effect function;[5,1]

$$\psi(\bar{h}) = 1 \pm \tau_1^2 + \frac{1}{2}\tau_1^4 \pm \frac{3}{4}\tau_1^6 + \dots$$

where the superscripts correspond to motion above the screen, and the subscripts to motion under the free surface, $\tau_1 = \sqrt{4\bar{h}^2 + 1} - 2\bar{h}$.

$\psi(\tilde{h}, \lambda, H)$ is the function of the combined effect of the boundary, aspect ratio and elasticity:

$$\psi\left(\tilde{h}, \lambda, H\right) = \frac{1}{1 - \operatorname{sign} RH\bar{A}_1 \frac{\lambda}{2} A_2}, \tag{22}$$

or

$$\psi\left(\tilde{h}, \lambda, H\right) = \frac{1}{1 - \operatorname{sign} R \frac{\bar{A}_1 H \frac{a_\infty \psi(\bar{h})}{2}}{\frac{\pi}{2} + \frac{a_\infty \psi(\bar{h})}{2\lambda}\zeta}}. \tag{23}$$

In an unbounded fluid, when $\psi(\bar{h}) = 1$, $\zeta = 1$, from (23) we set up the relation

$$\psi(\lambda, H) = \frac{1}{1 - \operatorname{sign} R \frac{\frac{1}{2}\bar{A}_1 H a_\infty}{\frac{\pi}{2} + \frac{a_\infty}{2\lambda}}}, \tag{24}$$

which for $a_\infty = 2\pi$ converts into the formula of Ref. 4:

$$\psi(\lambda, H) \approx \frac{1}{1 - \operatorname{sign} R \frac{0,75H}{\frac{1}{2} + \frac{1}{\lambda}}},$$

and for $\lambda \to \infty$ becomes the relation obtained in Ref. 3 for a thin elastic contour:

$$\psi(H) \approx \frac{1}{1 - \operatorname{sign} R \cdot 1,5H}$$

We express Eq. (21) as follows:

$$C_{y_{nd}} = C_{y_\infty} \psi(\bar{h}, \lambda), \tag{25}$$

where

$$C_{y_\infty} = a_\infty \alpha_0,$$

and

$$\psi(\bar{h}, \lambda) = \frac{\psi(\bar{h})}{1 + \frac{a_\infty \psi(\bar{h})}{\pi\lambda}\zeta\left(\tilde{h}\right)}. \tag{26}$$

In a bounded fluid for $\lambda \to \infty$, we obtain from formula (23) a representation for the function of the effect of the boundary and elasticity in a two-dimensional problem . /19

$$\varphi(\bar{h}, H) = \frac{1}{1 - \operatorname{sign} R \frac{\bar{A}_1 H a_\infty}{\pi} + F\left(\tau_1^2 - \frac{1}{4}\tau_1^4 + \frac{1}{4}\tau_1^6\right)}, \qquad (27)$$

which for $a_\infty = 2\pi$ converts into the result of Ref. 3:

$$\varphi(\bar{h}, H) \approx \frac{1}{1 - \operatorname{sign} R \cdot 1.5H + F\left(\tau_1^2 - \frac{1}{4}\tau_1^4 + \frac{1}{4}\tau_1^6\right)}.$$

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